the final condition

$$S(t_f) = 0$$

Note that very quickly

$$\dot{S} \simeq 0$$

Assuming[2,4] $S = 0$, Eq. (16) may be simply solved

$$S = \begin{pmatrix} \frac{2}{T}, & \frac{2}{T^2} \\ \frac{2}{T^2}, & \frac{4}{T^3} \end{pmatrix} \tag{17}$$

Equation (5) becomes

$$\begin{pmatrix} \dot{k}_1(t) \\ \dot{k}_2(t) \end{pmatrix} = \begin{pmatrix} \frac{2}{T}, & -1 \\ \frac{2}{T^2}, & 0 \end{pmatrix} \begin{pmatrix} k_1(t) - a(t) \\ k_2(t) \end{pmatrix} \tag{18}$$

Forming one second-order differential equation from the two coupled first-order equations yields

$$\ddot{k}_1(t) = (2/T)[\dot{k}_1(t) - a(t)] - (2/T^2)[k_1(t) - a(t)] \tag{19}$$

The solution in terms of the higher derivatives of the relative motion is,

$$k_1(t) = \sum_{n=0}^{\infty} [x^{(n+2)}(t) - Tx^{(n+3)}(t)]T^n \sum_{N=0}^{n} \frac{i^N}{(1+i)^n} \tag{20}$$

The control law, Eq. (3), becomes

$$\Delta a = (-2/T^2)x(t) - (2/T)v(t) - k_1(t) \tag{21}$$

It is interesting to note that the jerk term $x^{(3)}$ vanishes.

5. A Conjecture

Generally, all the higher derivatives of the relative motion are not known and the control laws would be truncated. Truncating at the order of the control, Eqs. (13) and (21) become

$$\Delta v(t) = (-1/T)[x(t) + Tv(t)] \tag{22}$$

and

$$\Delta a(t) = (-2/T^2)[x(t) + Tv(t) + (T^2/2)a(t)] \tag{23}$$

The truncated intercept laws[5,3] are

$$\Delta v(t) = [-1/(t_f - t)][x(t) + (t_f - t)v(t)] \tag{24}$$

and

$$\Delta a(t) = [-3/(t_f - t)^2]\{x(t) + (t_f - t)v(t) + [(t_f - t)^2/2]a(t)\} \tag{25}$$

In comparing these laws, it becomes apparent that Eqs. (22) and (23) are continuously trying to "intercept" at some fixed time increment, T, in the future. The possibility appears for a controller which attempts to track when supplied a "time constant" and to intercept when supplied the time to go. Does man behave in such a manner?[4,6]

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Evaluation of the Averaged Specular Component of Reflectance

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Introduction

RELIABLE predictions of radiant heat transfer require acceptable models for the radiation characteristics of surfaces. Since detailed knowledge of the radiation surface properties is generally not available for engineering materials, the analytical methods conventionally used for treating radiant heat exchange assume that the surfaces are either diffuse, specular or have diffuse plus specular components of reflectance.[1] In spite of the wide acceptance of the more realistic diffuse + specular model, a method for evaluating the specular component of reflectance in terms of known variables such as temperatures, reflectances, surface roughness and configuration has not been suggested. Two very simple approximate procedures[2,3] used in the past do not appear to be either adequate or in general correct.

In this Note, rigorous definitions of local as well as over-all (averaged over a surface) specular components of reflectance are presented. Approximations are then made to reduce the definitions to a workable form from which the over-all specular component of reflectance can be calculated and some illustrative results are also presented which examine the influence of various parameters.

Local Specular Component of Reflectance

The reflectance in general varies with location on the surface since the incident radiation field $I_{i\lambda}(\mathbf{r}, \theta', \phi')$ is a function of position vector $\mathbf{r}$ and incident direction θ', ϕ'. For an isotropic surface, the ratio of specular reflectance $\rho_\lambda{}^s$ to reflectance ρ_λ on the spectral basis is defined as

$$\frac{\rho_\lambda{}^s(\mathbf{r})}{\rho_\lambda(\mathbf{r})} = \frac{\int_{\phi_1'}^{\phi_2'} \int_{\theta_1'}^{\theta_2'} \rho_\lambda{}^s(\theta') I_{i\lambda}(\mathbf{r}, \theta', \phi') \cos\theta' \sin\theta' \, d\theta' \, d\phi'}{\int_{\phi_1'}^{\phi_2'} \int_{\theta_1'}^{\theta_2'} \rho_\lambda(\theta') I_{i\lambda}(\mathbf{r}, \theta', \phi') \cos\theta' \sin\theta' \, d\theta' \, d\phi'} \tag{1}$$

If the incident radiation field $I_{i\lambda}$ and the directional reflectances are known, Eq. (1) can readily be evaluated.

The importance of various parameters on $\rho_\lambda{}^s/\rho_\lambda$ is examined by assuming that the specular component of directional reflectance can be approximated by the Beckmann model[4]

$$\rho_\lambda{}^s(\theta', \sigma/\lambda) \simeq \rho_\lambda(\theta') g(\theta', \sigma/\lambda) = \rho_\lambda(\theta') \exp\{-[4\pi(\sigma/\lambda)\cos\theta']^2\} \tag{2}$$

where $\rho_\lambda(\theta')$ is the directional reflectivity of a smooth material with finite conductivity and σ is the rms roughness. Correcting for finite conductivity in this manner does not introduce any appreciable error and is justified on the basis of experimental evidence.[5] For a special case when $I_{i\lambda}$ and ρ_λ (or material of

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infinite conductivity) are independent of the incident direction θ', ϕ' substitution of Eq. (2) into Eq. (1) and integration yields

$$\rho_\lambda^s/\rho_\lambda = (e^{-b\cos^2\theta_2'} - e^{-b\cos^2\theta_1'})/b(\cos^2\theta_1' - \cos^2\theta_2') \quad (3)$$

with $b = [4\pi(\sigma/\lambda)]^2$.

Since in practical calculations, not only the directional, but also the spectral effects are important, Eq. (1) must be integrated over the entire spectrum. As a specific illustration consider that the incident intensity is independent of the azimuthal angle ϕ' and results from a black body at temperature T. The ratio $\rho_\lambda^s/\rho_\lambda$ averaged over the entire spectrum can then be expressed as

$$\frac{\rho^s}{\rho} = \frac{\int_0^\infty \left(\int_{\theta_1'}^{\theta_2'} I_{b\lambda}(T)\rho_\lambda(\theta')g(\theta', \sigma/\lambda)\, d(\sin^2\theta') \right) d\lambda}{\int_0^\infty \left(\int_{\theta_1'}^{\theta_2'} I_{b\lambda}(T)\rho_\lambda(\theta')\, d(\sin^2\theta') \right) d\lambda} \quad (4)$$

Some specific results for a perfectly reflecting material $\rho_\lambda(\theta') = 1$ [or $\rho_\lambda(\theta') = \text{const}$] presented in Fig. 1 and those not presented (for high-conductivity materials) show that $\rho_\lambda^s/\rho_\lambda$ and ρ^s/ρ are relatively insensitive to the directional distribution of the incident radiation field. The spectral and directional effects appearing in the function $g(\theta', \sigma/\lambda)$ are most important and those appearing in reflectance are relatively unimportant. This observation should be appropriate for metallic surfaces which in general have high reflectance. This finding is significant in that the previous studies[2] have shown that the choice of the model for the radiation characteristics is most critical for high-reflectance materials such as gold.

Over-All Specular Component of Reflectance

In practical design calculations, is neither ϕ' independent of θ' nor does the spectral distribution of intensity correspond to a definite temperature, as was assumed in the preceding discussion. The integration angles θ' and ϕ' are dependent purely on the configuration studied while the incident radiation field depends not only on the system but also on the temperature, radiation characteristics and roughness of the surfaces. The over-all (averaged over a surface) specular component of reflectance of surface i which forms an enclosure consisting of n surfaces is defined as

$$\overline{\left(\frac{\rho^s}{\rho}\right)_i} = \frac{\int_0^\infty \sum_{j=1}^n \int_{A_i}\int_{A_j} I_{j\lambda}(\mathbf{r}_j, \theta_j, \phi_j)\rho_{i\lambda}(\theta_i')g_i(\theta', \sigma/\lambda)K_{ij}\, dA_j\, dA_i\, d\lambda}{\int_0^\infty \sum_{j=1}^n \int_{A_i}\int_{A_j} I_{j\lambda}(\mathbf{r}_j, \theta_j, \phi_j)\rho_{i\lambda}(\theta_i')K_{ij}\, dA_j\, dA_i\, d\lambda} \quad (5)$$

where K_{ij} is the configuration kernel.

The intent here is to approximate the spectral intensity leaving surface $j(I_{j\lambda})$ in some gross manner so that $\overline{(\rho^s/\rho)_i}$ can be evaluated in a simple way. Following Bevans and Edwards,[6] we express $I_{j\lambda}$ as a sum of emitted and some mean reflected intensity. This appears to be justifiable since it has already been demonstrated that the specular component of reflectance is not very sensitive to the directional distribution of the radiation field.

In an enclosure consisting of highly reflecting surfaces, the contribution of emission is a small fraction of the leaving energy and hence can be ignored. Also, because of a large number of interreflections the radiant energy can be assumed to be well mixed spectrally. Assuming that $\rho_\lambda(\theta') = \text{const}$ and taking the spectral distribution to be the same as the spectral distribution of energy emitted into the enclosure, Eq. (5) can be written as[7]

$$\overline{\left(\frac{\rho^s}{\rho}\right)_i} = \frac{\int_0^\infty \left(\sum_{k=1}^n \varepsilon_{k\lambda} E_{bk\lambda} A_k\right) \sum_{j=1}^n w_j g_{ij\lambda} F_{ij}\, d\lambda}{\left(\sum_{k=1}^n \varepsilon_k E_{bk} A_k\right) \sum_{j=1}^n w_j F_{ij}} \quad (6)$$

where ε is the emittance, E_b is the black body emitted flux, F_{ij} is the configuration (view) factor, w_j are proportioned to the

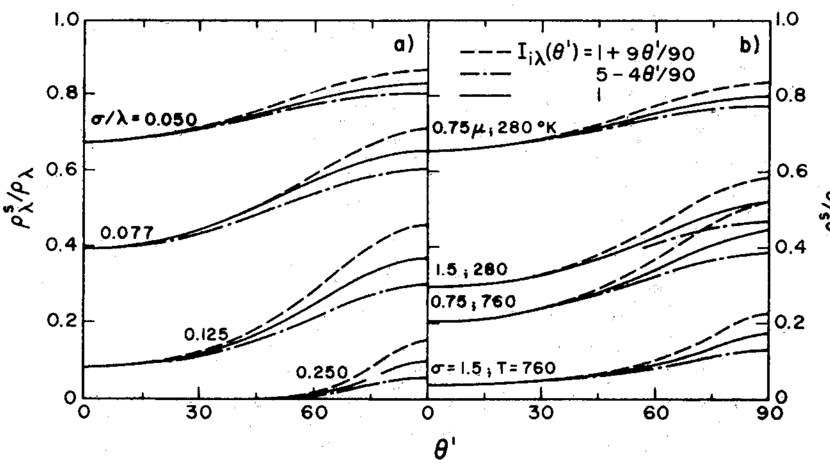

Fig. 1 Effect of directional variation of incident intensity on the specular component of reflectance for a perfectly reflecting material ($\rho_\lambda = 1$): a) spectral and b) total.

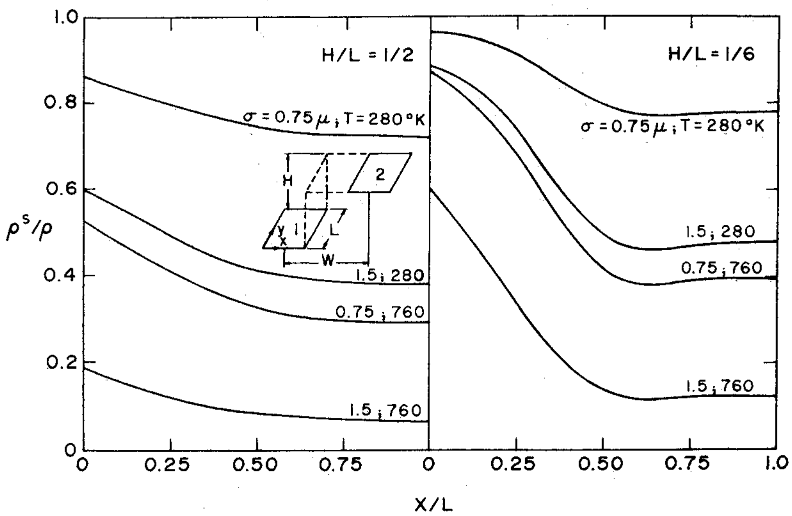

Fig. 2 Local specular component of reflectance of a perfectly reflecting surface for $y/L = _{11}$ with $W/L = 1.0$. The intensity of radiation leaving surface 2 is diffuse and uniform with the spectral distribution corresponding to blackbody emission at the indicated temperature.

Table 1 Variation of over-all specular component of reflectance for perfectly reflecting surface 1[a]

H/L	$\overline{(\rho^s/\rho)}_1$ $\sigma = 0.75\mu$ 280°K	590°K	760°K	$\sigma = 1.5\mu$ 280°K	590°K	760°K	$I_{2\lambda}$ $a,\ b$
	0.785		0.419	0.497		0.150	1, 19
$\frac{1}{6}$	0.774	0.500	0.394	0.474	0.202	0.131	1, 0
	0.769		0.383	0.465		0.122	20, −19
	0.764		0.361	0.446		0.101	1, 19
$\frac{1}{2}$	0.747	0.433	0.332	0.419	0.148	0.087	1, 0
	0.737		0.315	0.403		0.079	20, −19

[a] In this table, $W = L$; the intensity of radiation leaving surface 2 is diffuse and varies linearly with x, $I_{2\lambda}(x/L, y/L) = a + b(x/L)$. The spectral distribution corresponds to blackbody emission at the indicated temperature.

energy leaving each surface and

$$g_{ij\lambda}\pi A_i F_{ij} = \int_{A_i}\int_{A_j} g_i(\theta_1', \sigma/\lambda)K_{ij}\,dA_j\,dA_i \tag{7}$$

Note that in writing Eq. (7) it was assumed that $I_{j\lambda}$ is uniform over each surface. If this assumption is not valid, the surface can be subdivided into a number of smaller zones. For a highly reflecting enclosure, the weighting factors w_j may be justifiably assumed to be equal because of a large number of interreflections. In other cases, only those w_j need be retained for which surfaces A_j make the most contribution to $\overline{(\rho^s/\rho)}_i$.

In the other limiting situation when the enclosure consists of highly emitting surfaces, Eq. (5) can be expressed as[7]

$$\overline{(\rho^s/\rho)}_i = \int_0^\infty \sum_{j=1}^n E_{bj\lambda}(\varepsilon_{ji\lambda} g_{ij\lambda}\rho_{ij\lambda})F_{ij}\,d\lambda \Bigg/ \int_0^\infty \sum_{j=1}^n E_{bj\lambda}(\varepsilon_{ji\lambda}\rho_{ij\lambda})F_{ij}\,d\lambda \tag{8}$$

where the term in brackets represents the geometric thermal radiation characteristics[6] defined analogously to Eq. (7). In practical problems, it is convenient to evaluate these characteristics at some mean value of the angles. As a guide, if the distance between surfaces is more than five times the largest dimension of the source surface the error in applying the inverse square law is less than 1% (Ref. 8). For the special case when $\rho_\lambda(\theta') =$ const evaluation of ε and g at some mean angle θ_{ji} (the angle formed by normal to A_j and line joining A_j with A_i) can be expressed as

$$\overline{(\rho^s/\rho)}_i = \sum_{j=1}^n F_{ij}\int_0^\infty \varepsilon_{j\lambda}(\theta_{ji})g_i(\theta_{ij}', \sigma/\lambda)E_{bj\lambda}\,d\lambda \Bigg/ \sum_{j=1}^n F_{ij}\int_0^\infty \varepsilon_{j\lambda}(\theta_{ji})E_{bj\lambda}\,d\lambda \tag{9}$$

Example

The quantitative effects of various parameters on $\overline{(\rho^s/\rho)}_i$ are illustrated in Fig. 2 and Table 1. The configuration considered is shown as an insert in Fig. 2 and consists of two square surfaces. For highly reflecting material, the results were essentially the same. The trends were similar for other directional distributions of $I_{2\lambda}$. The results are given only for $y/L = \frac{7}{16}$ since for $y/L = \frac{1}{16}$ the difference was small.

Comparison of the results (including those not presented here) show that the local as well as the over-all component of reflectance is not sensitive to the variation in intensity leaving surface 2. The local specular component of reflectance shows a large variation across the surface for a relatively close configuration ($H/L = \frac{1}{6}$), especially for the large roughness and higher temperature. In spite of the large local variations, the over-all values (Table 1) are very close to the local values at locations where most of the energy is incident in near normal directions. This is due to the fact that the intensity incident at oblique angles has only a small contribution to the over-all specular component of reflectance. Use of the specular component of reflectance calculated in the manner presented in this Note yielded reasonably good agreement between predictions and data.[7]

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Trajectory Determination from Shock Arrival Times

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AN object traveling in air at a speed greater than that of sound generates a shock wave. Under ideal conditions, if the velocity is constant, the wave has the shape of a right circular cone except in the neighborhood of the object. In this Note a method will be given for calculating the position of the object at any instant and its vector velocity from measurements of the times at which the shock wave arrives at some microphones of known locations. Solutions to this problem have been

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